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The Radiation Characteristics of VLF Horizontal Wire Antennas over Multi-layered Earth

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Abstract. In this paper, the influence of the earth resistivity characteristics on the radiant field excited by a very low frequency(VLF) horizontal wire antenna is studied, and a 1D isotropic model of the horizontal layered earth is established. Theoretical formulas for the apparent resistivities of the layered ground and the electromagnetic field of antennas are derived for the case of observation points being on both of the site of antennas and far areas. The effects of different earth resistivity and multilayered earth on the radiation performances of the VLF horizontal antenna are simulated and analyzed. From the derivations and analyses, it is pointed out that the VLF horizontal antennas built on a region of high resistivity is conducive to radiant electromagnetic field effectively.

Keywords: multi-layered earth, horizontal wire antenna, radiation field

1. Introduction

In the VLF radio engineering, it is feasible to use horizontal wire antennas as transmitting antennas. The research shows that horizontal antennas connecting ground and the earth form a current loop, which can be equivalent to a loop antenna, and the radiation field is proportional to the square root of the effective resistivity for earth[1-3]. The greater the earth resistivity, the higher the antenna radiation efficiency, and the radiation efficiency of the VLF horizontal wire antenna is severely restricted by its erection site. Seeley, Peder, King and others proposed to load the antenna with inductance or form a multi-element array, which can effectively improve the radiation efficiency of the antenna [4-6]. Amin H and Al Ka'bi pointed out that the adaptive antenna arrays using cross-dipole antennas provide better performance [7]. Yan Chen, Hu Yang, and Taolin Liu propose a reflective array antenna based on incident fields that can improve antenna efficiency[8].

Due to environmental temperature, pressure and other factors, the antenna transmitting site can be divided into a horizontal multi-layer structure macroscopically according to the conductive properties of the medium. As early as 1970, G.E. Webber and others analyzed the effects of the ice thickness, complex permittivity, and earth conductivity on the amplitude and phase of VLF electromagnetic waves radiated by long horizontal antennas in the near region [9-10]. Subsequently, many scholars had conducted more research on the electromagnetic field generated by the horizontal electric dipole in the layered region, and these findings are well summarized in two works by Wait and King et al [11-12]. Chen H T, Mei, J. P and Popovic B. D and others have derived the formula for the electromagnetic field of the horizontal wire antenna above lossy half-space, and analyzed the trapped surface wave and the lateral wave [13-15]. However, theoretical studies of VLF horizontal antennas on horizontal layered ground are still lacking. In recent years, Jinhong Wang et al. have discussed the effects of frequency, sea depth, seafloor conductivity and receiver height on the fields and analysed the electromagnetic field excited by HEDs in the seawater dielectric layer[16]. The radiation pattern of HED on a four-layer medium is discussed by Zhu Hong Lin et al[17].

In this paper, the method of equivalent ground resistivity in literature [3] is used to theoretically study the apparent resistivity of the horizontally stratified ground. The expression of the antenna current and

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radiation efficiency are derived to better understand the effect of multi-layered earth on VLF horizontal antennas.

2. Methodology

2.1. Apparent Resistivity of the Horizontally Layered Earth

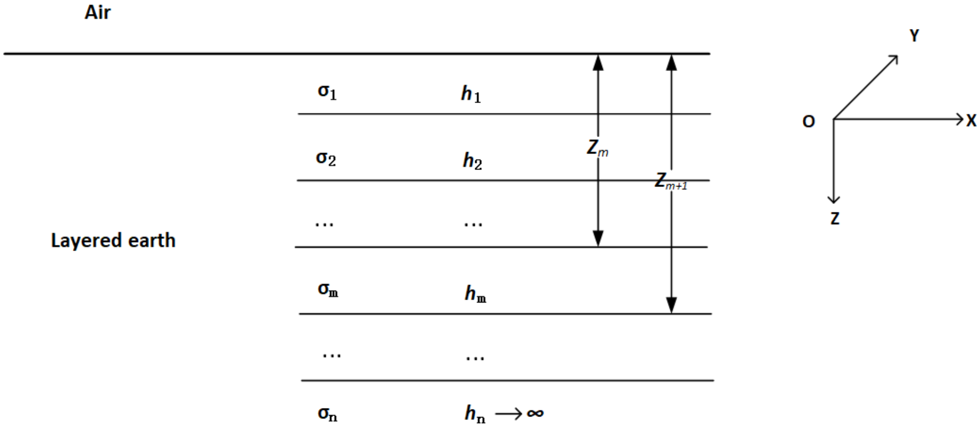


Fig. 1: One-dimensional isotropic layered model of horizontally layered earth.

The stratified model of the ground is showing in Fig.1. This figure indicates that the earth is layered in the vertical direction, and every layer is uniform and isotropic. Region I is characterized by conductivity σ_i, relative permittivity ε_{ri}, relative permeability μ_{ri}, and thickness of $h_i - h_{n-1}$, where $i = 1, 2, 3, \cdots, n$. Then, for common non-magnetic media, the wave numbers in the regions are:

$$k_0 = \omega\sqrt{\mu_0\varepsilon_0} \tag{1}$$

$$k_i = \omega\sqrt{\mu_0\left(\varepsilon_0\varepsilon_{ri} + j\sigma_i / \omega\right)} \quad i = 1, 2, 3, \cdots, n \tag{2}$$

The electric and magnetic fields can be derived easily via the Maxwell equations approach as:

$$E_x(z) = A_m e^{-jk_m z} + B_m e^{jk_m z} \tag{3}$$

$$H_y(z) = -\frac{k_m}{\omega\mu}\left(A_m e^{-jk_m z} - B_m e^{jk_m z}\right) \tag{4}$$

The characteristic impedance η_{0m} and wave impedance η_m of the m-th layer medium can be written as follow:

$$\eta_{0m} = -\frac{\omega\mu}{k_m} \tag{5}$$

$$\eta_m(z) = \frac{E_x(z)}{H_y(z)} = -\frac{\omega\mu}{k_m}\frac{A_m e^{-jk_m z} + B_m e^{jk_m z}}{A_m e^{-jk_m z} - B_m e^{jk_m z}} \tag{6}$$

Therefore, the impedance at the top of the layer can be expressed as:

$$\eta_m = -\frac{\omega\mu}{k_m}\frac{A_m e^{-jk_m Z_m} + B_m e^{jk_m Z_m}}{A_m e^{-jk_m Z_m} - B_m e^{jk_m Z_m}} = \eta_{0m}\frac{e^{-jk_m Z_m} + \dfrac{B_m}{A_m}e^{jk_m Z_m}}{e^{-jk_m Z_m} - \dfrac{B_m}{A_m}e^{jk_m Z_m}} \tag{7}$$

and the impedance at the bottom of the layer can also be written as:

$$\eta_{m+1} = \eta_{0m} \frac{e^{-jk_m Z_{m+1}} + \frac{B_m}{A_m} e^{jk_m Z_{m+1}}}{e^{-jk_m Z_{m+1}} - \frac{B_m}{A_m} e^{jk_m Z_{m+1}}} \tag{8}$$

From (7) and (8), we can get:

$$\frac{B_m}{A_m} = \left(\frac{\eta_{m+1} - \eta_{0m}}{\eta_{m+1} + \eta_{0m}} \right) e^{-2jk_m Z_{m+1}} \tag{9}$$

Then it is easy to get the impedance of the interface between the m-th layer and the (m+1)-th layer, (7) is equivalent to:

$$\begin{aligned} \eta_m &= \eta_{0m} \frac{e^{-jk_m Z_m} + \left(\frac{\eta_{m+1} - \eta_{0m}}{\eta_{m+1} + \eta_{0m}} \right) e^{(-2jk_m z_{m+1} + jk_m z_m)}}{e^{-jk_m Z_m} - \left(\frac{\eta_{m+1} - \eta_{0m}}{\eta_{m+1} + \eta_{0m}} \right) e^{(-2jk_m z_{m+1} + jk_m z_m)}} \\ &= \eta_{0m} \frac{1 + \left(\frac{\eta_{m+1} - \eta_{0m}}{\eta_{m+1} + \eta_{0m}} \right) e^{-2jk_m h_m}}{1 - \left(\frac{\eta_{m+1} - \eta_{0m}}{\eta_{m+1} + \eta_{0m}} \right) e^{-2jk_m h_m}} \\ &= \eta_{0m} \frac{\eta_{m+1} + \eta_{0m} th\left(jk_m h_m \right)}{\eta_{0m} + \eta_{m+1} th\left(jk_m h_m \right)} \end{aligned} \tag{10}$$

It can be seen from the above formula that the wave impedance of the interface between any two layers will be affected by the lower layer Therefore, to find the impedance of the interface between the air and the earth, that is, the surface impedance of the multi-layered earth, it must be calculated from the "infinitely deep" layer. Where, the characteristic impedance of the bottom medium is regarded as the impedance of the interface between it and the upper medium. The surface impedance is obtained by recursing upward layer by layer, which is also called apparent impedance η_0. Accordingly, its resistivity is the apparent resistivity ρ_0 (or apparent conductivity σ_0).

$$\rho_0 = \frac{1}{\omega \mu_0} \left| \eta_0 \right|^2 \tag{11}$$

$$\sigma_0 = 1 / \rho_0 \tag{12}$$

2.2. Radiation Efficiency of Low Altitude Antennas at Very Low Frequency Levels

The model of a horizontal wire antenna located above a planar ground is shown in Fig. 1. The current of the VLF horizontal antenna had been investigated by Chen in [11]. I_0 is the contribution of the excitation source which can be considered as the main part of the antenna current. I_+ and I_- are the scattered part of the antenna current, representing the reflection contribution from the left and right terminals of the antenna, respectively. The current can be written as follow:

$$\mathrm{I}\left(\mathrm{x}\right) = \mathrm{I}_0 e^{-jk_L|x|} + \mathrm{I}_+ e^{-jk_L x} + \mathrm{I}_- e^{jk_L x} \tag{13}$$

Where, k_L is the complex wavenumber of the antenna current, and its expression is given in the literature [6].

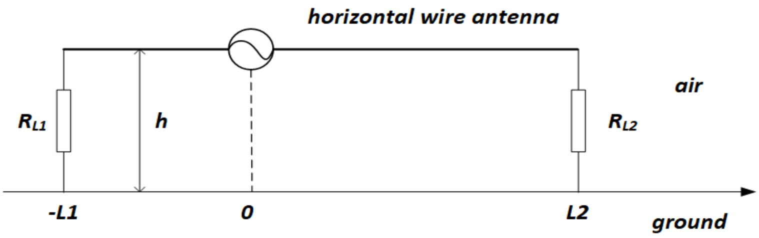


Fig.2 Horizontal wire antenna located above the earth

The radiation efficiency of a VLF horizontal antenna refers to its efficiency of radiating vertically polarized waves. It can be defined as the ratio of the input power of the single-amplitude lossless vertical antenna to the input power of the horizontal antenna when these antennas radiate the same vertically polarized electric field. The input power of a lossless vertical monopole is as follows:

$$P_{in}^{v}=\frac{160\pi^2\omega\varepsilon_0\cos^2\varphi}{\lambda^2\sigma_0}\left|\int_{-L_1}^{L_2}I(x)e^{jk_0x\cos\varphi}dx\right|^2 \tag{14}$$

where σ_0 is the apparent conductivity calculated by expression (11).

The input power of the horizontal antenna is:

$$P_{in}^{h}=R_{in}I_{in}^2 \tag{15}$$

I_0 is the input current, which can be obtained by the ratio of the input voltage V_{in} to the input impedance R_{in}.

$$I_{in}=\frac{V_{in}}{R_{in}} \tag{16}$$

Through the transmission line principle, the calculation formula of the input impedance R_{in} can be obtained:

$$R_{in}=R_c\frac{R_{L1}+R_c\tan(k_LL_1)}{R_c+R_{L1}\tan(k_LL_1)}+R_c\frac{R_{L2}+R_c\tan(k_LL_2)}{R_c+R_{L2}\tan(k_LL_2)} \tag{17}$$

$$R_c=\frac{60k_L}{k_0}\ln\left(\frac{2h}{a}\right) \tag{18}$$

In these formulas, R_c is the characteristic impedance of the VLF horizontal antenna, h is the erection height of the horizontal antenna, and a is the radius of the antenna.

Therefore, the radiation efficiency of the horizontal line antenna is:

$$\eta=\frac{P_{in}^{v}}{P_{in}^{h}}=\frac{160\pi^2\omega\varepsilon_0\cos^2\varphi}{\lambda^2\sigma R_{in}I_{in}^2}\left|\int_{-L_1}^{L_2}I(x)e^{jk_0x\cos\varphi}dx\right|^2 \tag{19}$$

By using equations (13) and (19), the general formula for the radiation efficiency of a VLF horizontal antenna can be written as:

$$\eta=\frac{160\pi^2\omega\varepsilon_0\cos^2\varphi}{\lambda^2\sigma_0R_{in}I_{in}^2}\left|\frac{1-e^{-j(k_LL_1+k_0\cos\varphi L_1)}}{j(k_L+k_0\cos\varphi)}I_0+\frac{e^{-j(k_LL_2+k_0\cos\varphi L_2)}-1}{j(-k_L+k_0\cos\varphi)}I_0+\right.$$
$$\left.\frac{e^{-j(k_LL_2+k_0\cos\varphi L_2)}-e^{j(k_LL_1-k_0\cos\varphi L_1)}}{j(-k_L+k_0\cos\varphi)}I_++\frac{e^{j(k_LL_2+k_0\cos\varphi L_2)}-e^{-j(k_LL_1-k_0\cos\varphi L_1)}}{j(k_L+k_0\cos\varphi)}I_-\right|^2 \tag{20}$$

3. Influence of Horizontal Layered Ground on VLF Horizontal Wire Antenna

3.1. Influence of Different Earth Resistivity on VLF Horizontal Antenna

The formula for the attenuation of the electric field distribution under the antenna with the depth of the ground is $E = E_0 e^{-\beta z}$, where E_0 is the electric field on the earth's surface, z is the depth of the ground, and β is the attenuation constant, that is the imaginary part of the wave number k. The horizontal wire antenna is erected on uniform ground with different resistivities. The antenna parameters are set as follows: the mid-point feeding,the operating frequency is $f = 20$ kHz, the antenna radius is $a = 0.6$ m, the antenna length is $L_1 + L_2 = 15$ Km, and the antenna erection height is $h = 10$ m. The earth resistivity is $500\,\Omega \cdot m$, $1000\,\Omega \cdot m$ and $3000\,\Omega \cdot m$ respectively, the relative permittivity is 10 and the magnetic permeability is 1. As shown in Fig. 3, the earth resistivity is constant, the electric field intensity near the surface is greater than that in the deep underground, and also decays exponentially with the increase of the underground depth. In addition, the higher the earth resistivity, the stronger the surface field strength, and the stronger the electric field strength at the same subterranean depth.

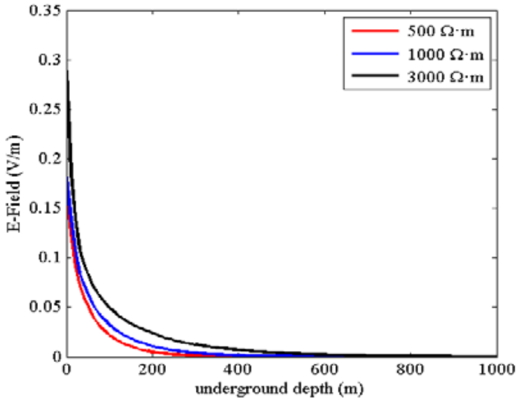
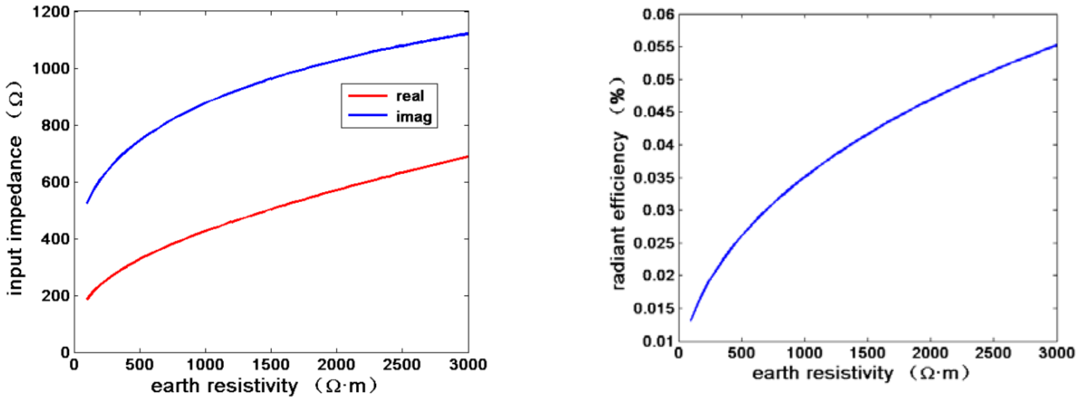


Fig.3 Electric field strength values at different underground depths

The VLF horizontal antennas fed at the earth of different resistivity are considered. Antenna parameter are the same as above. The result of input impedance and radiation efficiency of the antennas are shown in Fig. 4 respectively. As the resistivity of the ground increases, the input impedance and radiation efficiency of the antenna also increase.



（a） Input resistance （b） Radiation efficiency

Fig. 4 Antenna characteristics with different earth resistivity

3.2. Influence of Eight-Layered Region on VLF Horizontal Antenna

According to the actual situation, the earth surface can be divided into three categories: river water, soil and bare rock Then, three geological models of the antenna erection site are established. It is assumed that the antenna erection site is composed of eight layers of homogeneous geology, the relative permittivity is 10, the magnetic permeability is 1, the thickness of the first seven layers is 30m, and the eighth layer is infinitely deep. After the underground depth exceeds 100m, the geological structure and electrical parameters are basically the same. Therefore, the dielectric parameters of the fifth to eighth layers of the three models are set to be the same and the geological equivalent resistivity of river model is the smallest and that. The antenna parameter are the same as above.

Model 1: The surface layer of the first layer is river water with a resistivity of $10\,\Omega \cdot m$; the geological layer of the second layer is a river bed with a resistivity of $200\,\Omega \cdot m$; the geological resistivity of the third layer is $800\,\Omega \cdot m$; the geological resistivity of the fourth layer is $1500\,\Omega \cdot m$; the geological resistance of the fifth layer is $2200\,\Omega \cdot m$; The geological resistivity of the sixth layer is $2400\,\Omega \cdot m$; the geological resistivity of the seventh layer is 2600Ω · m; the geological resistivity of the eighth layer is $2800\,\Omega \cdot m$.

Model 2: The first layer is soil with an average resistivity of $300\,\Omega \cdot m$; the second layer is $600\,\Omega \cdot m$; the third layer is $1000\,\Omega \cdot m$; the fourth layer is $2000\,\Omega \cdot m$; the fifth to eighth layers. The geological resistivity of the fifth to eighth layers is the same as that of model one.

Model 3: The geological resistivity of the first layer is bare rock with a resistivity of $600\,\Omega \cdot m$; the geological resistivity of the second layer is $1000\,\Omega \cdot m$; the geological resistivity of the third layer is 1500 $\Omega \cdot m$; the geological resistivity of the fourth layer is $2000\,\Omega \cdot m$; The geological resistivity of the fifth to eighth layers is the same as that of model one.

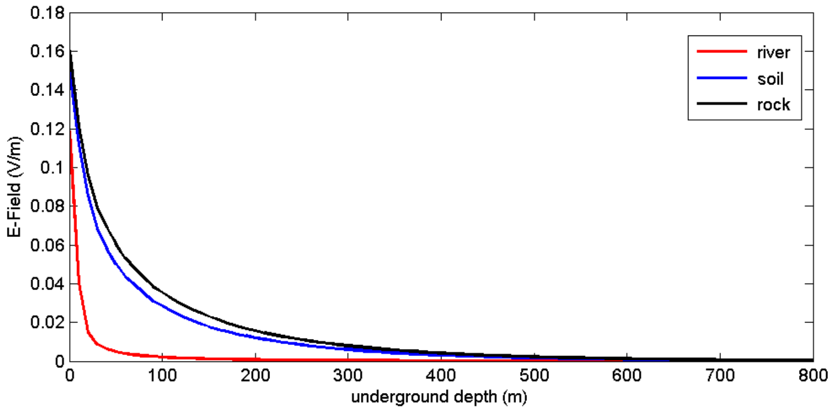


Fig. 5 Variation relationship of underground electric field intensity with different surface structures

The relationship between the underground electric field intensity and depth of the three models was simulated and calculated. As shown in Figure 5, the greater the resistivity of the earth, the stronger the electric field intensity near the surface, and the stronger the electric field intensity at the same underground depth. The smaller the surface resistivity is, the stronger the electric field near the surface is than that in the deep underground, the effective area of the current loop formed by the underground current and the antenna will be reduced, and the magnetic moment of the equivalent current loop will be smaller. Therefore, the electric field strength decreases rapidly with increasing subterranean depth.

Table 1: Radiation field strength and radiation efficiency of the three models

	Model 1	Model 2	Model 3
Electric field strength	4.33968 × 10-5	1.57980 × 10-4	2.23726 × 10-4
Radiation efficiency	0.54%	1.95%	2.76%

According to the model parameters, the radiation field strength and radiation efficiency of each model at a distance of 50 km were calculated. From the calculation results in Table 1, it can be seen that the VLF horizontal transmitting antenna erected on the bare rock with the highest resistivity has the best radiation ability, and the antenna erected on the river has the worst radiation ability. Therefore, when selecting the antenna erection field area, the site resistivity should be as high as possible, and the site surface resistivity should also be as high as possible.

4. Conclusion

The performance of the VLF horizontal wire transmitting antenna is closely related to the apparent resistivity of the ground at the site where the antenna is erected. From a macroscopic point of view, the ground resistivity naturally presents a horizontal layered form. From the above research and analysis, it can be concluded that the input impedance, current distribution, radiation field strength and other characteristics of the VLF wire antenna are affected by the erection site. The higher the surface resistivity is, the stronger the electric field near the surface is than that in the deep underground, the effective area of the current loop formed by the underground current and the antenna will increase, and the radiation efficiency will be greatly improved.

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